Nucleon polarizability and long range strong force from $\sigma_{I=2}$ meson exchange potential

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Abstract

We present a theory for how nucleon polarizability could be used to extract energy from nucleons by special electromagnetic conditions. A presentation of an experiment that validates the theory is presented. Also an new theory for a long range strong force is introduced by enhance the role of the $\sigma_{I=2}$ meson in nucleon nucleon potential made from mixed isospin σ meson.

Introduction

The theory about the possibility that nuclear effects can happen at energies lower than expected come with two problems:

- how to extract energy without having a large amount of radiation
- how to do this over a long range.

To solve those problems the first thing to do is to set the scale of low energy.

Low energy means that there is not enough energy to push nucleons close enough together to have strong interaction between them. The interaction that are left are the electromagnetic interaction with photons and electrons. For a single nucleon the theory that is used for this is nucleon polarizability i.e. how does the internal structure of the nucleon change with the interaction of photons.

The part of nucleon polarizabilities that will be investigated are the part where the polarizability takes energy from the nucleon. The second part links the special condition from polarizability where energy is extracted from the nucleon with the binding parts of nucleon-nucleon interaction. The third part has examples on how to create the special electromagnetic fields that are needed.

In the fourth part the long range problem is solved by introduce electron shielding of $\sigma_{I=0}$ meson. In short words the idea is that the short range properties of the strong force is due to massive exchange particles. To find long range parts of the nucleon force a search for mesons with mass less or equal to zero is done. For the strong force this is true for the $\sigma_{I=2}$ resonance found in

$\pi\pi$ scattering in the isospin 2 channel. The reason this long range part is not seen in normal nucleon-nucleon interaction is because isospin is not an exact symmetry and $\sigma_{I=0}$ and $\sigma_{I=2}$ are mixed in the interaction. Since the electron does not has any isospin charge the interaction with $\sigma_{I=0}$ is enhanced, which leaves the $\sigma_{I=2}$ meson free to interact over an long range.

Experiment

The experimental inputs have two source one is isotopic shifts observed in the experiments done in Lugano [2] and by Parkhomov [3], the other source is the observation of a charge neutral plasma with current running through it done in Doral at march 2017. To start with the isotopic shifts here it is notable is that most stable nucleon have a ground state of $0^+$and all seen receiver nucleons are of this state. The main seen reactions are:

$^{62-x}Ni + xn^* \rightarrow ^{62} Ni$

$^{27}Al + p^* \rightarrow ^{28} Si$

where p^* and n^* means a bound nucleon which has a source in another nuclide. Other possible reaction that might have been seen is

$^{58}Ni + 2n^* \rightarrow ^{60} Ni$

$^{7}Li + p^* \rightarrow ^{8} Be \rightarrow 2\alpha$

The main sources of the bound nucleons are

$^{64}Ni \rightarrow ^{62} Ni + 2n^*$

$^{7}Li \rightarrow ^{6} Li + n^*$

$^{55}Mn \rightarrow ^{54} Cr + p^*$

Other possibilities are:

$^{7}Li \rightarrow\ ^{6}He + p^*$

$^{7}Li \rightarrow\ ^{4}He + p^* + 2n^*$

$^{7}Li \rightarrow\ ^{4}He + p + 2n^*$

REPORT OF THE EXPERIMENT MADE ON MARCH 7 2017 IN THE LABORATORY OF THE FACTORY OF LEONARDO CORPORATION

Address of the site: 7861 NW 46th St., Doral, Florida, 33139 USA

Participants to the experiment: Carl-Oscar Gullström, Dr Andrea Rossi

Description of the apparatus

The circuit of the apparatus is made by a power source to supply direct current, a load made a 1 Ohm resistance, a reactor containing two nickel rods with LiAlH4 separated by 1.5 cm of space.

Measurements:

During the test a direct current was switched on and off. When the current was switched on a plasma was seen flowing between the two nickel rods. The current was running through the plasma but the plasma was found to be charge neutral from a Van Deer Graaf test. This implies that the plasma has an equal amount of positive ions flying in the direction of the current and negative ions(electrons) in the opposite direction.

Input: 0.105 V of direct current over a 1 Ohm resistance.

Energy output: The wavelength of the radiations out of the reactor has been measured by a spectrometer (Stellar Net spectrometer 350-1150 nm) and was integrated with the value of 1100 nm (1.1 microns).

The temperature of the surface of the reactor (a perfect black body) has been calculated with Wien's equation: $2900/\lambda$ (micron) $= 2900/1.1 = 2636$ K

By Boltzman Equation the effect is: $W = \sigma \times \epsilon \times T^4 \times A$

A $= 1.0$ cm^2

$\epsilon = 0.9$

By substitution: $W = 5,67 \times 10^{12} \times 0.9 \times 4.8 \times 10^{13} = 244.9$

Nucleon Polarizability

The main theory for nucleon polarizability used here is baryon chiral perturbation theory [1]. Other theories that exist are heavy baryon chiral perturbation theory and fixed-t dispersion theory which both have polarizability constants within the same ranges as baryon ChPT. For a review see for example [6]. The theory of polarizability is carried out by first taking the ground state E_0 of the nucleon and then perturb the N-γ interaction with an effective Hamiltonian:

$$H = E_0 - H_{eff}$$

where H_{eff} is the effective Hamiltonian. If the condition $H_{eff} < 0$ is fulfilled then the new state is an excitation and $H_{eff} > 0$ corresponds to a binding energy which could be used to extract energy out of the nucleon. The effective Hamiltonian used is[5]:

$$H^{(2)}_{eff} = -\frac{1}{2} 4\pi \left(\alpha_{E1} \bar{E}^2 + \beta_{M1} \bar{H}^2 \right)$$

$$H^{(3)}_{eff} = -\frac{1}{2} 4\pi \left(\gamma_{E1E1} \bar{\sigma} \cdot \bar{E} \times \dot{\bar{E}} + \gamma_{M1M1} \bar{\sigma} \cdot \bar{H} \times \bar{H} - 2\gamma_{M1\dot{E}2} E_{ij} \sigma_i H_j + 2\gamma_{E1M2} H_{ij} \sigma_i E_j \right)$$

$$H^{(4)}_{eff} = -\frac{1}{2} 4\pi \left(\alpha_{E1\nu} \dot{\bar{E}}^2 + \beta_{M1\nu} \dot{\bar{H}}^2 \right) - \frac{1}{12} 4\pi \left(\alpha_{E2} E^2_{ij} + \beta_{M2} H^2_{ij} \right)$$

where α_x, β_x, γ_x are polarizability constants, E and H are components of the electromagnetic fields. σ is the Pauli spin matrices of the nucleon and E_{ij} is given by $E_{ij} = \frac{1}{2} \left(\nabla_i E_j + \nabla_j E_i \right)$ with the same relation for H_{ij}. i and j stands for space indexes.

For the polarizability constants with and even number of perturbation the experimental values and theoretical predictions are shown in table 1. Since every part of the EM field is a square the only time the condition $H_{eff} > 0$ is

	α_{E1}	β_{M1}	α_{E2}	β_{M2}	$\alpha_{E1\nu}$	$\beta_{M1\nu}$
Proton						
BχPT Theory[1]	11.2 ± 0.7	3.9 ± 0.7	17.3 ± 3.9	-15.5 ± 3.5	-1.3 ± 1.0	7.1 ± 2.5
Experiment(PDG[9])	11.2 ± 0.4	2.5 ± 0.4				
Neutron						
BχPT Theory[1]	13.7 ± 3.1	4.6 ± 2.7	16.2 ± 3.7	-15.8 ± 3.6	0.1 ± 1.0	7.2 ± 2.5
Experiment(PDG[9])	11.8 ± 1.1	3.7 ± 1.2				

Table 1: Theoretical and experimental values of the proton and neutron static dipole, quadrupole and dispersive polarizabilities. The units are $10^{-4}\ fm^3$ (dipole) and $10^{-4}\ fm^5$ (quadrupole and dispersive).

	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}	γ_0	γ_π
Proton						
BχPTTheory[1]	-3.3 ± 0.8	2.9 ± 1.5	0.2 ± 0.2	1.1 ± 0.3	-0.9 ± 1.4	7.2 ± 1.7
MAMI 2015[7]	-3.5 ± 1.2	3.16 ± 0.85	-0.7 ± 1.2	1.99 ± 0.29	-1.01 ± 0.13	8.0 ± 1.8
Neutron						
BχPT Theory[1]	-4.7 ± 1.1	2.9 ± 1.5	0.2 ± 0.2	1.6 ± 0.4	0.03 ± 1.4	9.0 ± 2.0

Table 2: Theoretical and experimental values of the proton and neutron static spin polarizabilities. The units are $10^{-4}\ fm^4$.

fulfilled when the polarizability constant is negative. For that only theoretical values exists the strong polarizability constant of the magnetic quadrupole and the weaker electric dispersive polarizability constant.

The odd number of perturbation is called spin polarizability and has no meaning in a classic EM field. This means that the interaction is due to the non standard electromagnetic parts of the nucleon ie the strong force. To find the conditions $H_{eff} > 0$ the situation is a bit more complicated. Since the field includes the nucleon spin, the positive value condition depends on the alignment of the nucleon spin with the EM field parts. An interesting and easy accessible by experiment part is the case of forward and backward scattering. The polarizability constants are labeled γ_0 and γ_π. They are related to the spin polarizability constants by:

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

$$\gamma_\pi = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

The spin polarizability experiment and theoretical values are displayed in table 2. The binding condition are fulfilled for the proton for example by the γ_0 constants and there is a possibility to define regions where $H_{eff} > 0$ while the four spin polarizability constants are known.

Special electromagnetic fields The special electromagnetic fields that are required for extracting energy out of a nucleon is a magnetic quadrupole or a changing electric field. An important factor is that $H_{eff} > 0$ should be done for the total reaction. Otherwise the total interaction corresponds to an excitation. This means that only the center of an magnetic quadrupole is relevant where there is no dipole field. For the electric dispersive polarizability this is a problem since a time derivative always should increase or decrease a field not to have a nonzero component which implies that the dipole polarizability should be dominant at some point except for extremely fast oscillating fields with a weak strength.

For spin polarizability the special condition are easiest seen from the source of the spin polarizability [10]:

$p = \gamma \nabla \left(\bar{S} \bullet \bar{B} \right)$

where p is the polarization, B is the magnetic field and S is the spin of the nucleon. From this one can deduct that to extract energy from spin polarizability a magnetic field must go through the spin axis of nucleon.

Link to nucleon-nucleon force in nuclides

Since polarizability is a temporary interaction state extracting energy out of the nucleon, by it is also temporary. Permanent energy extraction out of nucleon could be found in fusion of nucleons into nuclides. Then to make the temporary energy exctraction permanent, one needs to move the polarizability interaction into nucleon-nucleon interaction. To do this a link between to nucleon-nucleon interaction and γ/e-nucleon interaction has to be done to see that the polarizability interaction sets the nucleon in the same state that it is when its bound in a nuclide. A second motivation for the link is to establish a range of spin polarizability. If this polarizability uses the exchange particles of the strong force, the range (which is motivated by the absent of spin polarizability in a classical EM field) one would expect the interaction range to be similar to the one of the strong force.

The main difference between nucleon-γ/e polarizability interaction and strong force nucleon-nucleon interaction is that the photon and electron do not have isospin and hence those interaction should be suppressed in polarizability compared to nucleon-nucleon interaction.

From the first attempt to do nucleon-nucleon interaction of yukawa[11] using a pion as an heavy exchange particles, today the best nucleon nucleon interactions are found by using quantum Monte Carlo methods [12]. The operators used there are not only meson exchange but also Δ baryon. It includes 2N and 3N forces with more than one exchange particle and fits parameters by Monte Carlo methods. Each term in the Hamiltonian is a multiplication of operators including space, spin, isospin and orbits of the nucleons with a fitted coupling strength. The link binding of polarizability is found for those operators that is equal to the one corresponding to exchange that gives $H_{eff} > 0$ for polarizability and gives a binding term in the Hamiltonian of N-N interaction. Other models exist, for example the Bonn model[13] is more described with one boson exchange and links each operator with a meson exchange.

The condition $H_{eff} > 0$ from polarizability is found when magnets are opposite and separated on the z axis. A match for the binding polarizability constants are the tensor and spin orbit operators. In the tensor operator the scalar product of space and total spin is non zero. The operator used is $S_{12} = 2\left(\frac{3(\bar{s}\cdot\bar{r})}{r^2} - \bar{s}\cdot\bar{s}\right)$ which in one boson exchange model corresponds to η exchange. S_{12} is nonzero only when the spin spin coupling are in triplet state and a magnetic quadrupole are then only present when s_z is opposite. The second operator is the spin orbit coupling L*S which corresponds to σ meson exchange. This operator has a back to back magnet from the fact that one of the magnets comes from the spin and the other from the orbit. Notable are that the σ meson is not a well established particle and instead s-wave scattering of two pions are usually describing this interaction. For the binding terms of dispersive polarizability the operator needs to include a time operator. That is also found in spin orbit coupling since the L terms includes a time derivative.

A further example that N-N bindings give a magnetic quadrupole is the deuteron. The deuteron is in a spin 1 state but since the proton and neutron have opposite magnetic g factors the magnetic dipoles are opposite. Calculation of the deuteron magnetic moment and binding energy also includes a nonzero tensor force which sets the space separation on the same axis as the spin.

Suggestion of EM states

From the link between spin/quadrupole polarizabilities and binding interaction in nuclides one finds that the possibilities to extract energy from polarizability interaction with $H_{eff} > 0$ need to use the short range interaction of strong force. This arises from the fact that the operator match uses heavy exchange particles, the σ and η mesons. Also unless free nucleon with the energy level of bound exist there need to be electrons to hold the nucleon in the special temporary bound state. Further motivation for the use of electrons or virtual photons over real photons is that a real photon always has an electric and magnetic dipole field component so that the scalar polarizabilities are stronger than the binding ones. However as seen in the next part special condition of polarizabilities creates a long range strong force. This means that after an initial phase the need for nearby electrons disapear.

The electromagnetic interactions that are left are bound electrons in atoms which have a wave function that overlaps with the core. These are especially s orbitals but also d orbitals have an overlap with the core. Next limit on the special needed electromagnetic states are the fact that it needs to be able to extract energy while still stay near the core. This means that no kinetic energy could be released that separate the electron from the core except if a long range strong force are already present. The way that energy still is extracted is done by hyper-fine splitting of the energy levels. For d state elctrons the fine splitting of energy levels should be a possible source. The important part for this to work is that the electrons needs to be unpaired in order to make a spin flip to emit photon in the microwave region.

Suitable elements that are naturally in 1S states are the alkali and coin metals. In addition some more metals could be in 1s states this includes nickel, platinum, niobium, molybdenum, ruthenium, rhodium and chrome. A d_{z^2} state is particularly good since this is a magnetic quadrupole seen from the nucleon.

Binding polarizability is seen both in spin polarizability and magnetic quadrupole polarizability when two magnets are separated on the z axis with opposite ends against each other. This is a problem for free s-state electrons because the space separation of the nucleon spin with the electron is in the x-y plane with the magnetic moments on the z-axis. A strong magnetic field would tilt the magnetic dipoles but align them to have equal orientation. The special conditions of opposite magnetic dipoles on the dipole axis is only fulfilled for an s-state electron in the center of an external magnetic quadrupole. The nucleon and electron must then be placed on each side of the center to have opposite magnetic fields. An example of a free external magnetic quadrupole is seen if an s-state is seen as a projection on the z-axis of a d-state electron. Then in the d_{z^2} state the angular momentum of the electron works as a magnetic quadrupole for the nucleon.

Other external magnetic quadrupoles could be found by looking on magnetic properties of materials. In a metal, small magnets are found in grains of several atoms which have magnetic momentum aligned in the same direction. If the metal is ferro or permamagnetic the magnetic order of the grains is aligned on the magnetic dipole axis which would create few to none magnetic quadrupoles. The strongest magnetic quadrupoles are then found in non permanent magnetic materials in between the grains. A good way to put atoms in between the grains is by using liquid metal embrittlement and/or hydrogen embrittlement. There charged ions of the liquid metal/hydrogen fills the space in between grains to make cracks and at the same time place them in strong magnetic quadrupoles.

With a nearby long range strong force an interaction between a current of free electrons and nuclides could use magnetic quadrupoles polarizabilities. For relativistic electrons the spin gets aligned with the direction of motion. In the absent of a magnetic field the spins are equal distributed between spin up and down. To create a magnetic quadrupole two electrons should be in an opposite spin state which corresponds to 50% of the electrons if paired togheter. Since the magnetic quadrupole pairs extract energy when interacting with the right nuclides their speed will be enhanced and they start to dominate over the pairs with aligned spins. This will turn an electron current into a charge neutral plasma with current running through it as seen in the Doral experiment.

Long range strong force

Putting a nucleon in a pure magnetic quadrupole could extract energy out of it but the effect is only temporary. When the nucleon gets out of place in a normal "neutral" EM environment, polarizability would absorb energy by scalar polarizability up to the ground state. To transfer nucleon over a long range, a long range potential of the strong force has to be established. The nucleon-nucleon interaction are short ranged because of massive exchange particles. The basic short range from Yukawa one pion exchange potential[11] is given by (without spin and isospin dependency):

$$\frac{g}{4\pi}\frac{e^{-m_\pi r}}{r}$$

where m_π is the pion mass and g is an Yukawa coupling. Compare this with the electromagnetic potential :

$$\frac{q}{4\pi}\frac{e^{-m_\gamma r}}{r}$$

where q is the charge of the source particle and $m_\gamma = 0$ gives the coulomb potential with space dependency of $1/r$.

A long range strong force is found if an exchange particle has a mass or rather squared mass equal below the photon mass:

$$m^2 \leq m_\gamma^2$$

Such a meson is found in $\pi\pi$ s-wave scattering in the isospin 2 channel. In nucleon-nucleon interaction $\pi\pi$ s-waves are found in 3N forces. The pion s-wave from the Illinois model is not accurate and only a plausible 1MeV size strength is included, but it is needed to explain energy levels of light nuclides. To move to the one boson exchange Bonn model the σ exchange potential there is given by:

$$V_{NN}^{(\sigma)}(r) = \int \frac{d^3q}{(2\pi)^3} e^{iqr} \frac{g_{\sigma NN}^2}{-q^2 - m_\sigma^2} = -\frac{g_{\sigma NN}^2}{4\pi}\frac{e^{-m_\sigma r}}{r}$$

The theory of $\pi\pi$ scattering could be found in [14]. This paper uses both fixed-t dispersion and ChPT as seen in polarizability theory. In $\pi\pi$ scattering the σ mass is related to scattering phase shifts. The phase shift equation is [15]:

$$tan\delta_l^I = \sqrt{1 - \frac{4m_\pi^2}{s}} q^{2l} \left\{ A_l^I + B_l^I q^2 + C_l^I q^4 + D_l^{\hat{I}} q^6 \right\} \left(\frac{4m_\pi^2 - s_l^I}{s - s_l^I} \right)$$

where q is the momentum transfer of the scattering pion, A...D are constants, l gives spin and I isospin channel. The kinematic variable s is given by $s = 4m_\pi^2 + q^2$ and s_l^I specifies where δ passes through 90^o. At lowest order the A constants equal the s-wave scattering length $a_0^0 = 0.22$ and $a_0^2 = -0.044$ ref??. The negative sign of a_0^2 is what gives the necessary $m^2 \leq m_\gamma^2$ relation. The parameter s_l^I is given by $s_0^0 = 36.77m_\pi^2$, $s_1^1 = 30.72m_\pi^2$ and $s_0^2 = -21.62m_\pi^2$.

The isospin 1 channel is isolated in a L=1 state by Dirac motivation of a total anti symmetric wave function and this pole corresponds to the ρ meson. Since isospin is not a perfect symmetry the L=0 channel is mixed and σ exchange in nucleon nucleon interaction includes both I=0 and I=2 σ mesons. The m_σ used in Bonn is 550 MeV while s_0^0 gives a mass at 844 MeV. Assuming equal mix of $\sigma_{I=2}$ and $\sigma_{I=0}$ in $NN\sigma$ coupling gives the more correct mass of m_σ =543 MeV.

To create a pure potential with $\sigma_{I=2}$ meson only the $\sigma_{I=0}$ has to be absorbed. Since the electron does not have an isospin component, e-N binding through nucleon polarizability is reduced. To get a size of the free $\sigma_{I=2}$ interaction one could start with γ vector meson couplings. In the quark model the coupling ratio is given by $g_{\omega\gamma} = 3g_{\rho\gamma}$ and $g_{\phi\gamma} = -\frac{3}{\sqrt{2}}g_{\rho\gamma}$. Assuming that two vector mesons is the strongest part of pion coupling, this gives a factor of two and σ from $\pi\pi$ scattering is also a factor of 2. The total enhancement of $\sigma_{i=2}$ potential in e-N coupling is a factor of $(g_{\omega\gamma}/g_{\rho\gamma})^4$ i.e. $3^4 = 81$. Using this suppression and assuming equal mix of isospin states, the σ mass becomes virtual with the value $-im_\sigma = 642$ MeV. The σ exchange s-wave scattering then changes the exponential term to an oscillating one.

The Illinois model gives a size of this interaction in 3N $\pi\pi$ s-wave terms. There the part of the binding energy is for ^4He 0.6 MeV and for ^7Li 0.9 MeV. Transfer nucleon with this potential should make the target nuclide change the electron with a nucleon and release kinetic energy at the same time. But s-wave states corresponds to 0^+ in the nuclides, so the target nucleus should stay in the ground state.

Since isospin comes with direction I_z that equals charge, the type of $\sigma_{I=2}$ matters for the potential. Proton has $I_Z = +1/2$ and neutron $I_Z = -1/2$ while Δ has $I_z = \pm 3/2$. From opposite attract properties of the potentials, protons are attracted to potential created by neutrons and neutrons to potentials created by protons. To create a full $\sigma_{I=2}$ meson NΔe is enhanced, compare to NNe where the Δ comes from the 3N force i.e. 3N-e interaction is most probable needed. In full 4N ground state clusters the N-N potential fulfills the binding polarizability conditions and e/γ-N interaction can not extract energy. This leaves 1 hole nuclides as the only source of $\sigma_{I=2}$ long range potentials. Examples of isotopes are ^7Li, ^{27}Al and in the case of attractive 3 neutron cluster ^{61}Ni. Since most free 3N states has 2 neutrons and 1 proton (except ^3He) the majority of $\sigma_{I=2}$ potentials are attractive to protons. Neutron attractive $\sigma_{I=2}$ potentials could be formed if a proton transfer does not form a perfect 4N state. For example the reaction Ni+p* with p* from manganese or lithium would give copper isotope below the ground state. The reaction would still be possible as a temporary unstable state with the aid of a proton attractive $\sigma_{I=2}$ potential from for example ^{27}Al. Neutron transfer should be stronger compare to proton transfer because no coulomb repulsion exist between the proton and the target nuclide.

Summary and discussion

The needed parameters are not known from experiment except for the spin polarizability constants. Also the long range potential from $\sigma_{I=2}$ is unknown for both detailed theory and experimental. To extract those constants experimentally a theoretical way would be to use $\pi\pi$-lepton scattering with a measurement of nucleon properties in a nearby region. Practically it is questionable if a pion beam with high enough luminosity is possible and to construct.

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